# Section 3.7: Derivatives of Inverse Functions

This section will explore the relationship between the derivative of a function and the derivative of its inverse.

## The Derivative of an Inverse Function

**Inverse Function Theorem**

Let be a function that is both invertible and differentiable. Let be the inverse of . For all satisfying ,

.

Alternatively, if is the inverse of , then

.

Media: Watch these [video](https://youtu.be/o94iS6NMz2M)1 and [video 2](https://youtu.be/YgucAop5yaM) examples on the derivative of inverse functions.

Examples

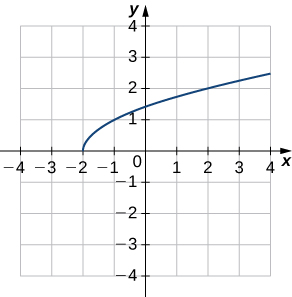
1. Use the inverse function theorem to find the derivative of . Compare the resulting derivative to that obtained by differentiating the function directly.



1. Find the derivative of by using the inverse function theorem.

Use the graph of to

* 1. Sketch the graph of , and
  2. Use part a. to estimate .



**Extending the Power Rule to Rational Exponents**

The power rule may be extended to rational exponents. That is, if is a positive integer, then

.

Also, if is a positive integer and is an arbitrary integer, then

.

Media: Watch this [video](https://youtu.be/HWFtcIRhr5c) example on derivatives of functions with rational exponents.

Example: Find the equation of the line tangent to the graph of at .

## Derivatives of Inverse Trigonometric Functions

**Derivatives of Inverse Trigonometric Functions**

Media: Watch these [video](https://youtu.be/Vy002NmyT5s)1 and [video2](https://youtu.be/yNYhNARIG50) examples on derivatives of inverse trig functions.

Examples

1. Find the derivative of the following functions:
2. The position of a particle at time is given by for . Find the velocity of the particle at time .

Media: Watch this [video](https://youtu.be/iGa83EYBk70) example on applications of derivatives of inverse trig functions.

1. A pole stands feet tall. An angle is formed when wires of various lengths of feet are attached from the ground to the top of the pole, as shown in the following figure. Find the rate of change of the angle when a wire of length feet is attached.

